A Generalized Model for Coupled Lines and its Applications to Two-Layer Planar Circuits

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***Abstract*—A generalized network model for asymmetrical and inhomogeneous coupled lines has been derived based on normal mode parameters. This model is useful for synthesis of single and multi-layer coupled-line circuits, such as planar baluns and directional couplers. The synthesis procedures are described and have been verified by comparing with analysis results.**

I. Introduction

UNIFORMLY coupled lines are widely used in filters, couplers and impedance matching networks at microwave frequencies. Analysis techniques for coupled lines have been reported extensively [1]–[4]. For synthesis of components using coupled lines, network models are needed. Several four-port network models have been reported [5], [6]. A network model of coupled lines in a homogeneous medium has been derived by Malherbe using capacitance matrices and its applications for network synthesis have been reported [5]. However, this model is not valid for coupled lines in an inhomogeneous medium where phase velocities of the two modes are not equal. In inhomogeneous media, not only the capacitance matrix but also the inductance matrix of coupled lines is needed for the derivation of network models. Another model has been proposed by Chang and Lee for inhomogeneous lines in the cases where the congruence condition is satisfied [6]. This condition is not generally true for two-layer circuits. Therefore, for the synthesis of general coupled-line circuits, a new network model is needed which does not make assumptions of congruence or homogeneity.

This paper presents a derivation of such a generalized network model for coupled lines. The derivation is based on the four-port impedance or admittance matrix which could be found by using normal mode parameters [4]. The final four-port network representation can be simplified for two-port coupled-line components by applying different termination conditions on the other two ports. These two-port equivalent networks are useful in the synthesis of two-port coupled-line circuits (like filters).

The four-port model reported in this paper has been used for the synthesis of a planar balun circuit [11]. By using this model for planar balun, a useful synthesis procedure has been developed. An example is presented and the results are verified by comparing with the direct analysis of the circuit.

Also, the proposed model has been used in the design of directional couplers [12], [13]. The conditions for a directional coupler with an infinite directivity or perfect match are given. General design equations are given for a directional coupler with specified input impedances and coupling. The final circuit can then be synthesized based on the normal mode parameters given by the design equations. Since the solutions are not unique, the designers have the freedom to choose between single or multi-layer and symmetrical or asymmetrical geometries. An example of two-layer directional coupler is included. Again, the design procedure is verified by comparing with analysis results.

II. Generalized Coupled-Line Models

A. Z-Matrix Based on Normal Mode Parameters

The characteristic parameters for the general case of an inhomogeneous, asymmetrical, uniformly coupled lines can be derived by solving the generalized coupled-transmission-line equations. Thus, coupled lines (Fig. 1) could be characterized by two modes (c mode and π mode) using normal mode parameters [4]. These parameters are the phase velocities (ν_c and ν_π), the ratios of the voltage on the conductor 2 to the voltage on the conductor 1 (R_c and R_π) and the line impedances (Z_{c1}, Z_{c2}, $Z_{\pi1}$ and $Z_{\pi2}$). If the coupled lines are symmetrical, the two modes correspond to the even and odd modes of excitation ($R_c = 1$, $R_\pi = -1$). For the homogeneous case, the two modes have the same phase velocity and therefore are not uniquely defined. One of the possible mode definitions in the homogeneous case is given in [12]. Generally, the normal mode parameters can be obtained directly from capacitance and inductance matrices. The Z-matrix for a section of two coupled lines in terms of these normal parameters is shown in Table I, where γ_c and γ_π are the propagation constants for the two modes of the coupled lines.

B. Network Model for a Section of Coupled Lines

Every element in the Z-matrix of Table I is a sum of two terms. The first one depends on the electrical length for c mode only and the other depends on the electrical

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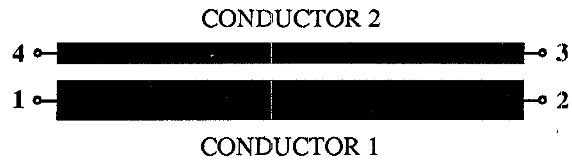

Fig. 1. Schematic of a section of uniformly coupled lines.

TABLE I
Z-MATRIX FOR A SECTION OF COUPLED LINES

$\frac{Z_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{Z_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$
$\frac{Z_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{Z_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$
$\frac{R_cZ_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_c^2Z_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_c^2Z_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$
$\frac{R_cZ_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_cZ_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_c^2Z_{c1}\,\text{csch}\,\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi1}\,\text{csch}\,\gamma_\pi l}{1-R_\pi/R_c}$	$\frac{R_c^2Z_{c1}\coth\gamma_c l}{1-R_c/R_\pi}+\frac{R_\pi^2 Z_{\pi1}\coth\gamma_\pi l}{1-R_\pi/R_c}$

length for π mode only. This Z-matrix can be separated into a sum of two matrices such that the elements in each one of them depend only on either θ_c or θ_π, the electrical lengths for the two modes. Each of these two matrices can be modeled by using one transmission line section and two transformers as shown in Fig. 2.

In this network (Fig. 2), the ideal transmission line section has its electrical length specified by one of the two modes and its line impedance depends on the characteristic impedance of that mode only. The turns ratio values for the two transformers are the ratio of voltages on the two conductors of the coupled lines for that mode. The second part of Z-matrix (Table I) is modeled by a similar network with R_c, R_π, Z_{c1}, and θ_c replaced by R_π, R_c, $Z_{\pi 1}$ and θ_π, respectively.

The final four-port network model for a coupled-line section is found by connecting these two networks in series. The result is shown in Fig. 3. If a Y-matrix representation is used instead of Z-matrix for the derivation of network model, another equivalent model (Fig. 4) can be derived by using a similar procedure. These two models are exact models of general uniformly coupled lines and they are equivalent to each other. These four-port models for coupled lines are useful for the synthesis of coupled-line circuits and their applications in the designs of planar baluns and directional couplers are given in Sections III and IV.

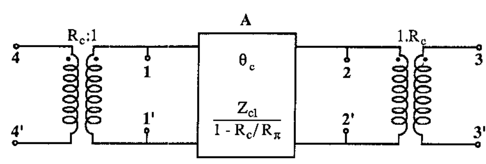

Fig. 2. Network model for the first part of Z-matrix in Table I. Block A is an ideal transmission line section with electrical length θ_c and characteristic impedance $Z_{c1}/(1 - R_c/R_\pi)$.

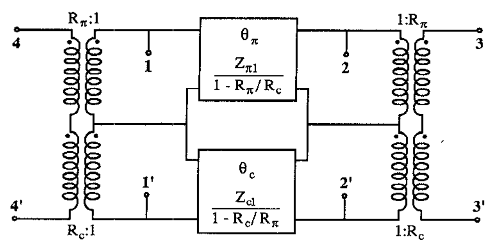

Fig. 3. Network model for general coupled lines, based on the Z-matrix.

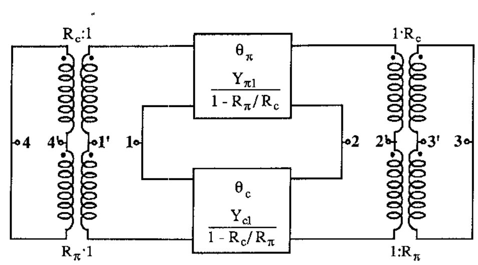

Fig. 4. Network model for general coupled lines, based on the Y-matrix.

C. Two-Port Network Models for Coupled-Line Sections

By application of different port conditions to the four-port models, the equivalent circuits of a large variety of coupled-line two-port structures (Fig. 5) can be obtained. Only six configurations are shown here. Similar models may be derived for other two-port coupled-line components. Some of these two-port models are of the same forms as those obtained by using matrix characterization of the two-port circuits [7]. However, we note that models for configurations (c) and (f) in Fig. 5 are different from those given in [7]. The corresponding two models in [7] make use of Richards transform with the inherent assumption that the two electrical lengths θ_c and θ_π are equal at the frequency the model is derived. Thus, the models derived in the present paper are more accurate when θ_c

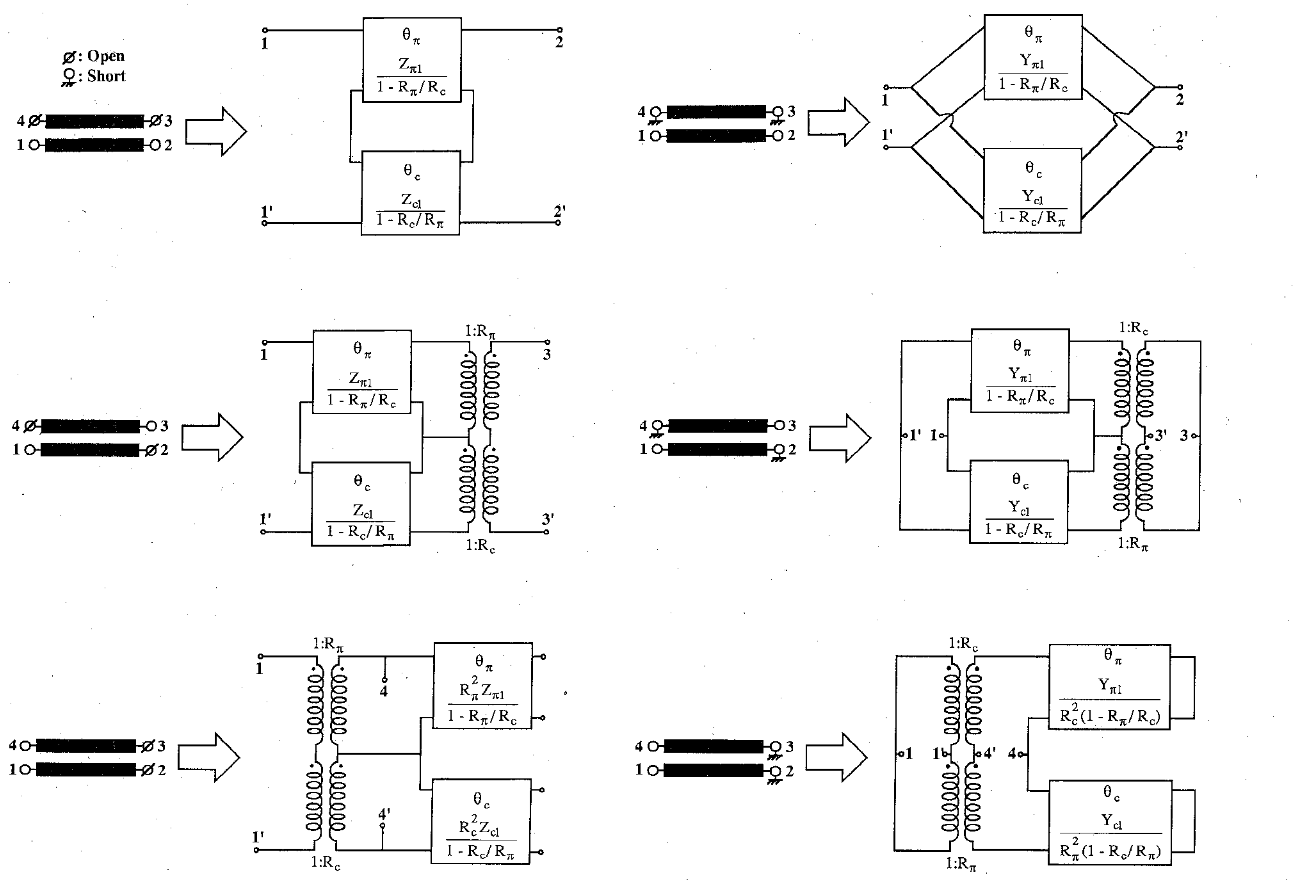

Fig. 5. Models for some two-port coupled line configurations.

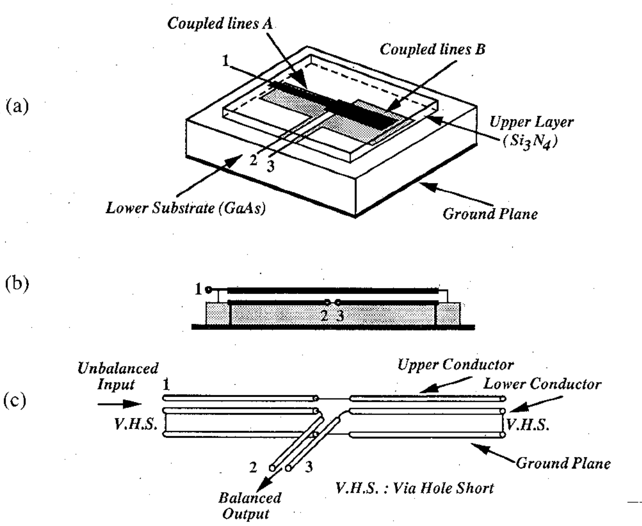

Fig. 6. (a) Three dimensional view of the planar balun; port 1 is the unbalanced port and terminals 2 & 3 constitute the balanced port, (b) the sectional view, and (c) the equivalent transmission line circuit.

and θ_π are not equal. Also, Zysman and Johnson [1] have reported models for similar two-port configurations for the case of symmetrical coupled lines. The more general models reported here reduce to their models for the case of symmetrical lines.

III. Synthesis of Planar Marchand Balun

A. Modeling of Planar Marchand Balun

The new four-port network model of coupled lines has been used for designing a two-layer monolithic balun configuration (Fig. 6) described in [11]. The unbalanced input of this balun is at terminal 1 fed by a microstrip at the upper metallization level. The balanced output is at terminals 2 and 3 at the lower metallization level. This kind of balun structure is similar but different from Marchand balun [8] in that the transmission line sections in different layers are not isolated from each other. The circuit could be viewed as the connection of two coupled-line sections (A and B in Fig. 6(a)). By using the new network model derived in Section II (the lower conductor and the upper conductor in Fig. 6 correspond to the conductor 1 and the conductor 2 respectively, in Fig. 1), the network representation of this planar balun is shown in Fig. 7. This network is reconfigurated by using network theory and expressed as shown in Fig. 8. This model may be compared with the equivalent circuit of the original Marchand balun [8]. We note that in this equivalent circuit of the planar balun, the transmission line sections have different electrical lengths, which is due to the inhomogeneity of the configuration, whereas in the equivalent circuit of Marchand balun they have the same electrical length. Another difference is that, in the planar balun, the transformers T_1,

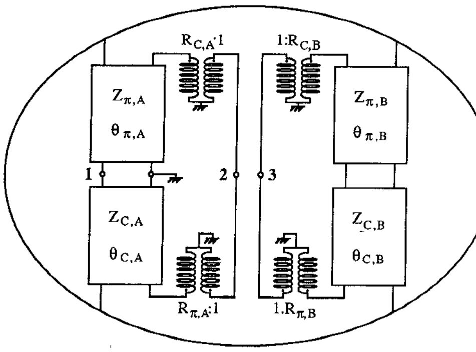

Fig. 7. Network model of the planar balun. $Z_c = Z_{c2}(1 - R_\pi/R_c)$, $Z_\pi = Z_\pi R_c^2(1 - R_\pi/R_c)$.

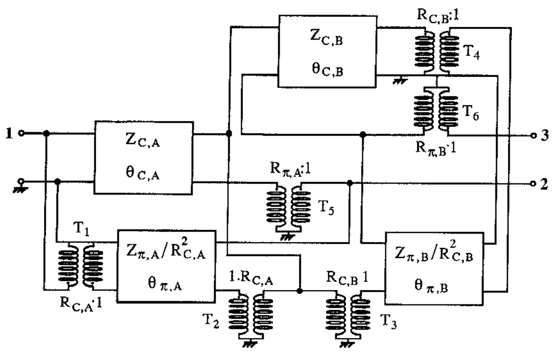

Fig. 8. Another equivalent circuit of the planar balun.

T_2, T_3 and T_4 in the equivalent circuit represent coupling between transmission line sections.

B. Synthesis Procedure

If the geometry of coupled lines is chosen such that $R_\pi \approx 1$, the transformers T_5 and T_6 in the equivalent network become 1 : 1 transformers and can be replaced by through connections. Also, when R_c is large, the other transformers can be replaced by the corresponding connections shown in Fig. 9. This results in a network like the equivalent circuit of the original Marchand balun (Fig. 9), which is more suitable for implementation of the synthesis procedure. In this figure, another transmission line section (Z_S, θ_S) is included which corresponds to the planar line at the balanced output port. To carry out synthesis based on network in Fig. 9, it is convenient to assume that all transmission line sections have equal electrical lengths so that the Richards' transform can be used. In a typical case discussed later, we note that there may be about 20% difference in the electrical lengths. However, as the example shows, the procedure developed is useful even in such a case. The synthesis procedures for such a network configuration are available in [9], [10].

C. Design Example

As an example, a two-layer planar balun with bandwidth of 4 to 24 GHz, the unbalanced input impedance of 25 Ω and the balanced output impedance of 50 Ω, has been designed using 6 mil GaAs ($\epsilon_r = 12.9$) substrate with a second dielectric layer of 1.8 μm thick silicon nitride ($\epsilon_r = 7.0$). The network model shown in Fig. 9 is synthesized by using the graphs given in [10], which are based on the synthesis procedure given by Horton and Wenzel [9]. The elements values are obtained from the graph for the fourth order balun with 50 Ω source resistance and then scaled down to one half for 25 Ω input impedance. This method yields:

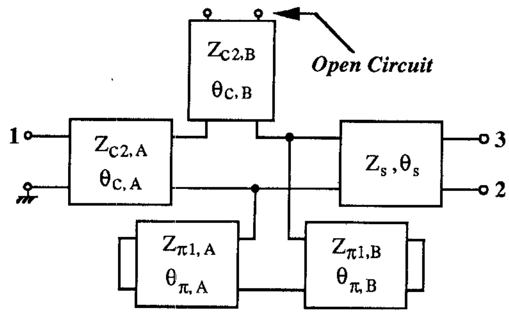

Fig. 9. Simplified model of the planar balun.

$$Z_{C2,A} = 31\ \Omega$$

$$Z_{C2,B} = 15\ \Omega$$

$$Z_{\pi 1,A} + Z_{\pi 1,B} = 80\ \Omega$$

$$Z_S = 39\ \Omega$$

These parameters are used to design the coupled-line sections and the results are:

Coupled Lines A	Coupled Lines B
$W_1 = 160\ \mu m$	$W_1 = 160\ \mu m$
$W_2 = 10\ \mu m$	$W_2 = 20\ \mu m$
$Z_{c2} = 29\ \Omega$	$Z_{c2} = 15\ \Omega$
$Z_{\pi 1} = 41\ \Omega$	$Z_{\pi 1} = 40\ \Omega$
$R_c = 41.78$	$R_c = 18.60$
$R_\pi = 0.98$	$R_\pi = 0.98$
$v_c = 1.28 \times 10^8$ m/s	$v_c = 1.24 \times 10^8$ m/s
$v_\pi = 1.02 \times 10^8$ m/s	$v_\pi = 1.02 \times 10^8$ m/s

where W_1 and W_2 are the line widths of the lower and the upper conductors, respectively. The length of each pair of coupled lines is determined by using the average value of the phase velocities for c mode and π mode and is quarter wavelength at 14 GHz. We note that, for both of the coupled lines A and B, $R_\pi \approx 1$ and R_c is large. This justifies the use of approximation made in implementation of the synthesis procedure.

The ideal responses of this balun are shown in Figs. 10 and 11. These results are based on following assumptions: (i) the propagation modes for each coupled line are quasi-TEM modes, (ii) there are no dielectric and ohmic losses, (iii) the electrical lengths for all transmission line

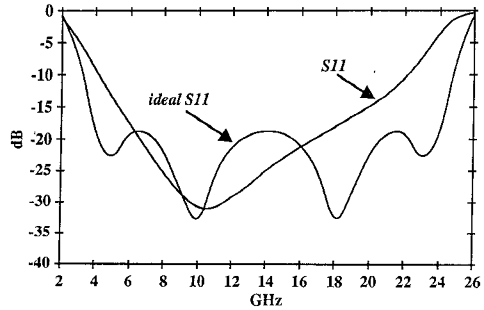

Fig. 10. Return loss at the input (unbalanced) port for the synthesized and ideal baluns.

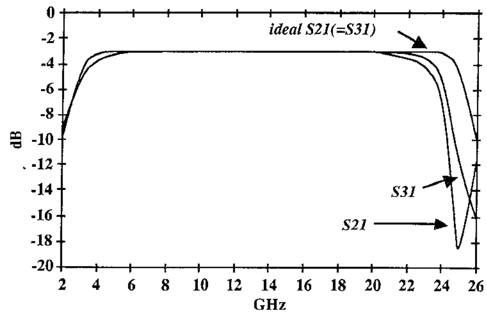

Fig. 11. Insertion loss for the synthesized and ideal baluns.

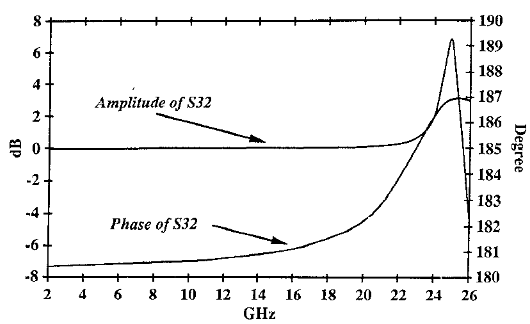

Fig. 12. Amplitude and phase unbalance at the output (balanced) port for the synthesized balun.

sections are equal, (iv) the transmission line formed by upper two conductors is isolated from the line formed by lower conductors and ground plane. Also, the various discontinuity reactances are not included. The responses of the synthesized circuit without the assumptions (iii) and (iv), based on the model in Fig. 8, are also shown as S_{11}, S_{21} and S_{31} in these figures for comparisons. The amplitude and phase at the two terminals of the balanced output port are compared in Fig. 12. It should be noted that dielectric and ohmic losses could be incorporated in the analysis by using complex phase constant for each transmission line section.

From these results, we note that the simplifications in the synthesis procedures (assuming $R_c \approx 1$, R_π large and equal electrical lengths for all of the transmission line sections) yield a network with slightly (about 10%) lower bandwidth than the ideal design. Still, the suggested procedure provides a useful synthesis method.

IV. Design of Directional Coupler

The design of directional couplers using symmetrical coupled lines is reported in [12]. For the homogeneous case, directional couplers using asymmetrical coupled transmission lines are also reported [13]. Using the new model of coupled lines in this paper, the properties of directional couplers using asymmetrical coupled lines in an inhomogeneous medium are investigated. A synthesis approach for general coupled-line directional couplers is also presented.

A. Condition for Infinite Directivity

Assume that the coupled-line section is terminated by impedances Z_{01} at port 1 and port 2 and by impedances Z_{02} at port 3 and port 4. It is shown in Appendix I that for perfect isolation ($S_{31} = 0$), the coupler needs to satisfy the following equation:

$$(Z_{01}Z_{02}Z_{c1} - R_c R_\pi Z_{c1} Z_{\pi 1}^2) \tan \theta_\pi = (Z_{01}Z_{02}Z_{\pi 1} - R_c R_\pi Z_{\pi 1} Z_{c1}^2) \tan \theta_c \qquad (1)$$

If the coupled lines are symmetrical and homogeneous (i.e., $R_c = 1$, $R_\pi = -1$ and $\theta_c = \theta_\pi$), then (1) is simplified as $Z_{01}Z_{02} = Z_{c1}Z_{\pi 1}$ and this is the only condition needed to be satisfied for perfect isolation at all frequencies. However, if the two modes of the coupled lines have different phase velocities, the isolation of this coupler depends on frequencies. It should be noted that if the average value of the two mode phase velocities is used as the phase velocity in the coupler design, at the frequency where the length of the coupler is quarter wavelength, tan $\theta_c \approx -\tan \theta_\pi$ (note that one of θ_c and θ_π is greater than $\pi/2$). This will reduce the equation (1) to $Z_{01}Z_{02} = R_c R_\pi Z_{c1} Z_{\pi 1}$. Since generally the value of $R_c R_\pi$ is negative, the condition for perfect isolation is not possible at the design frequency. However, as shown in an example later, good isolation can be obtained near the design frequency.

B. Conditions for Perfect Isolation and Perfect Match

It is shown in Appendix II that for perfect isolation and perfect match, the coupler needs to satisfy following two equations:

$$Z_{01} = \sqrt{Z_{c1}Z_{\pi 1} \frac{(|R_c|Z_{\pi 1} + |R_\pi|Z_{c1})}{(|R_c|Z_{c1} + |R_\pi|Z_{\pi 1})}} \qquad (2)$$

$$Z_{02} = \sqrt{R_c^2 R_\pi^2 Z_{c1}Z_{\pi 1} \frac{(|R_c|Z_{c1} + |R_\pi|Z_{\pi 1})}{(|R_c|Z_{\pi 1} + |R_\pi|Z_{c1})}} \qquad (3)$$

It should be noted that these two equations are the exact solutions for the homogeneous cases and they are the approximate solutions in the inhomogeneous cases under the assumptions of $Z_{\pi 1, c1} \tan \theta_{\pi, c} >> Z_{01}$ and $Z_{01} \tan \theta_{\pi, c} >> Z_{\pi 1, c1}$. The coupling from port 1 to port 4 is found to be

$$S_{41} = \sqrt{\frac{|R_c R_\pi|(Z_{c1} - Z_{\pi 1})^2}{(|R_c|Z_{\pi 1} + |R_\pi|Z_{c1})(|R_c|Z_{c1} + |R_\pi|Z_{\pi 1})}}. \quad (4)$$

Equations (2) to (4) may be used for the design of inhomogeneous, asymmetric couplers (including two-layer couplers). So far, the matching impedances for an asymmetrical coupled-line directional coupler have been determined by computer iterations [14], [15]. For this iterative procedure, one starts with terminating impedances $Z_{01} = \sqrt{Z_{c1}Z_{\pi 1}}$ and $Z_{02} = \sqrt{Z_{c2}Z_{\pi 2}}$. First, Z_{01} is optimized for minimum reflection at port 1 at the design frequency by keeping Z_{02} fixed. Then, Z_{01} is fixed to this new value, the new Z_{02} is found such that the reflection at port 4 is a minimum. This process is repeated until the reflection coefficients at all ports are minimized. Based on the work reported here, these terminating impedances can be determined directly using analytic solutions (2) and (3). As an example, the equations are used to determine the matched loads for a two-layer coupled-line directional coupler with the normal mode parameters as

$$R_c = 0.8359,$$

$$R_\pi = -3.1537,$$

$$Z_{c1} = 85.29\ \Omega, \text{ and}$$

$$Z_{\pi 1} = 10.29\ \Omega.$$

The port terminating impedances are obtained as

$$Z_{01} = 48.46\ \Omega \text{ and}$$

$$Z_{02} = 47.74\ \Omega$$

and the value of coupling in this case is

$$S_{41} = -2.88 \text{ dB}.$$

These values of Z_{01} and Z_{02} are the same as the final results of the computer iterations [14].

C. Synthesis Procedure

When the termination impedances (Z_{01} and Z_{02}) and the coupling (S_{41}) of a coupler are given, the normal mode parameters are determined by the following equations (these equations are derived from (33)–(37) in Appendix II, for the case when R_c is positive and R_π is negative):

$$R_\pi = -\sqrt{\frac{(1 + a)Z_{02}}{(1 + b)Z_{01}}} \quad (5)$$

$$R_c = \sqrt{\frac{b(1 + a)Z_{02}}{a(1 + b)Z_{01}}} \quad (6)$$

$$Z_{c1} = Z_{01}\sqrt{\frac{a(1 + b)}{(1 + a)}} \quad (7)$$

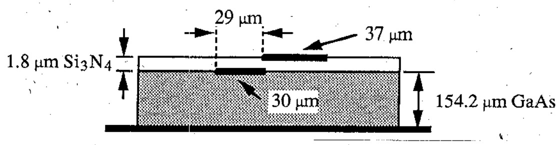

Fig. 13. Two-layer 3-dB coupler.

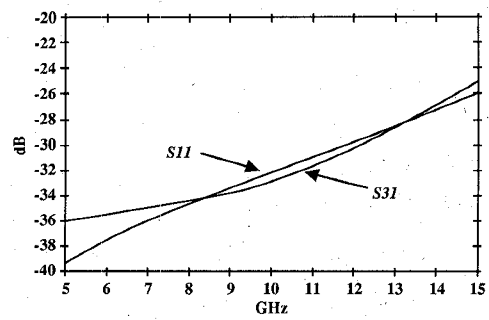

Fig. 14. Match and isolation of the synthesized 3-dB coupler.

$$Z_{\pi 1} = Z_{01}\sqrt{\frac{(1 + b)}{b(1 + a)}} \quad (8)$$

where the parameters a and b determine the coupling as

$$S_{41}^2 = \frac{(\sqrt{ab} - 1)^2}{(1 + a)(1 + b)}. \quad (9)$$

When S_{41} is given, the choices of parameters a and b are not unique. The different combinations of parameters a and b will result in different possible geometries of the coupler. For the special case of symmetrical couplers, parameters a and b are equal. As an example, let us consider the design of a 3-dB two-layer coupler with input impedances 50 Ω at all four ports. The value of parameter a is found by carrying out iterative computations until a physically realizable geometry is obtained. One such value of parameter a is 30. Then parameter b is found to be 2.1 by using (9). The normal mode parameters are calculated using (5) to (8), the results are:

$$R_c = 0.84$$

$$R_\pi = -3.16$$

$$Z_{c1} = 86.65\ \Omega$$

$$Z_{\pi 1} = 10.91\ \Omega$$

These normal mode parameters are realized on a 6 mil GaAs substrate with a second dielectric layer of 1.8 μm thick silicon nitride. The line widths are 30 μm and 37 μm for the lower and the upper conductors, respectively. The horizontal distance between these two lines is 29 μm as shown in Fig. 13. The analysis results justify this synthesis procedure and are shown in Figs. 14 and 15. We note that a 3-dB coupler with good performance (more than 30 dB isolation and input match) can be designed using this method.

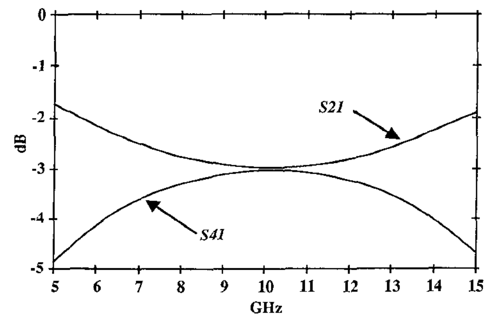

Fig. 15. Coupling and insertion loss of the synthesized 3-dB coupler.

V. Concluding Remarks

A new model for generalized uniformly coupled transmission lines in an inhomogeneous medium is derived. The model is useful for various planar coupled-line circuits. In this paper, the model has been successfully used for synthesis of two-layer monolithic planar baluns for which no synthesis procedure has been available so far. Also, the model has been used for the design of asymmetrical couplers in an inhomogeneous medium and the design equations are given. The modeling approach discussed in this paper is also applicable to other (two-layer) planar circuits such as re-entrant type couplers, Lange couplers and three-line balun configurations.

This paper has focused on synthesis procedures to yield normal model parameters of multiconductor transmission lines for a given balun or coupler performance. The problem of finding actual physical dimensions to yield these normal mode parameters has not been addressed here. A geometry synthesis procedure (based on iterative analysis) for this purpose is being developed currently.

Appendix I
Derivation of the Condition for Perfect Isolation

Assume that the port 4 is terminated by a load Z_{02} (Fig. 16) and the parameters Z_1 and Z_2 are defined as:

$$Z_1 \equiv \frac{V_1}{R_\pi I_4} = \frac{V_1 Z_{02}}{R_\pi (R_\pi V_1 + R_c V_2)} \tag{10}$$

and

$$Z_2 \equiv \frac{V_2}{R_c I_4} = \frac{V_2 Z_{02}}{R_c (R_\pi V_1 + R_c V_2)}; \tag{11}$$

then, the network model can be simplified as shown in Fig. 17. Similarly, we terminate the port 3 by a load Z_{02} and define Z_3 and Z_4 as

$$Z_3 \equiv \frac{V_3 Z_{02}}{R_\pi (R_\pi V_3 + R_c V_4)} \tag{12}$$

and

$$Z_4 \equiv \frac{V_4 Z_{02}}{R_c (R_\pi V_3 + R_c V_4)}. \tag{13}$$

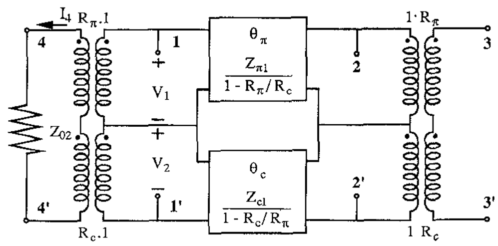

Fig. 16. Coupled line model with load Z_{02} at port 4.

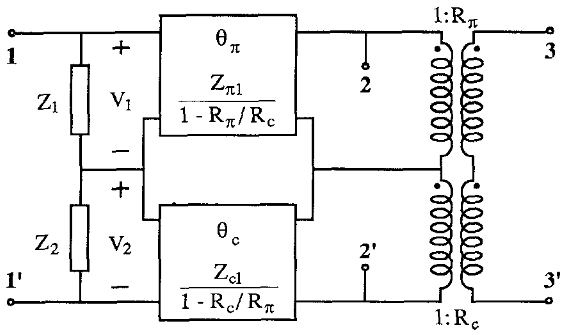

Fig. 17. Simplified model with two transformers and the load at port 4 replaced by Z_1 and Z_2.

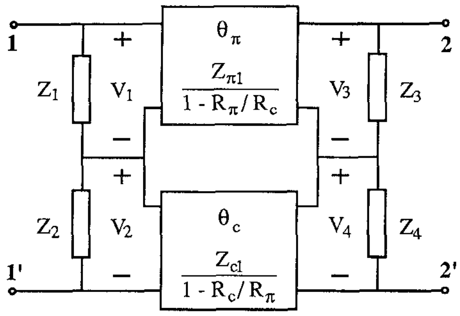

Fig. 18. Simplified model. All the transformers and loads at port 3 and port 4 are replaced by Z_1, Z_2, Z_3 and Z_4.

The two-port equivalent circuit of the coupled lines with port 3 and port 4 terminated as mentioned above is shown in Fig. 18. It should be noted that the impedance parameters Z_1, Z_2, Z_3 and Z_4 used in the above network are functions of V_1, V_2, V_3 and V_4. The voltages at four ports are

Port 1: $V_1 + V_2$

Port 2: $V_3 + V_4$

Port 3: $R_\pi V_3 + R_c V_4$

Port 4: $R_\pi V_1 + R_c V_2$

Now, for good isolation ($S_{31} = 0$), we require the voltage at port 3 to be zero, i.e.

$$R_\pi V_3 + R_c V_4 = 0. \tag{14}$$

This implies that the impedances Z_3 and Z_4 should be infinite. If the port 2 is terminated by load Z_{01}, the equivalent circuit reduces to the one shown in Fig. 19. By using

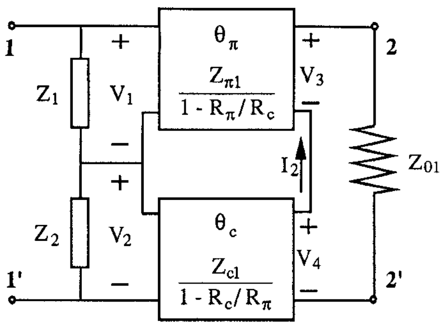

Fig. 19. Simplified model. Components Z_3 and Z_4 have been removed due to the condition of perfect isolation ($S_{31} = 0$).

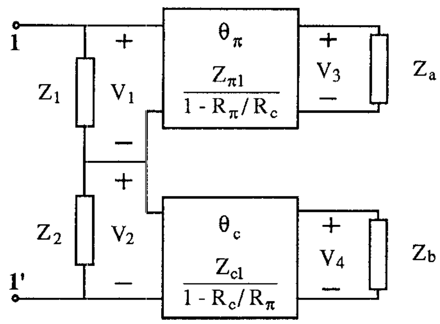

Fig. 20. Simplified model. The load at port 2 (Z_{01}) is separated as two impedances Z_a and Z_b.

Kirchhoff's voltage law at the right hand side of the equivalent circuit, we have another equation as following:

$$V_3 + V_4 = I_2 Z_{01}. \tag{15}$$

Using (14) and (15), we find the equivalent load impedances for the two transmission line sections in the above network as

$$\frac{V_3}{I_2} = \frac{R_c Z_{01}}{R_c - R_\pi} \equiv Z_a \tag{16}$$

and

$$\frac{V_4}{I_2} = \frac{R_\pi Z_{01}}{R_\pi - R_c} \equiv Z_b. \tag{17}$$

Here, Z_a and Z_b are not functions of V_1, V_2, V_3 and V_4. This is the result of imposing the condition that the voltage at port 3 is zero. The equivalent circuit is then as the one shown in Fig. 20. Now, we define $Z_{i\pi}$ and Z_{ic} as the input impedances looking into the transmission line from the left hand side of the transmission line sections. We have

$$Z_{i\pi} = \left(\frac{R_c Z_{\pi 1}}{R_c - R_\pi}\right) \frac{Z_{01} + jZ_{\pi 1} \tan \theta_\pi}{Z_{\pi 1} + jZ_{01} \tan \theta_\pi} \tag{18}$$

and

$$Z_{ic} = \left(\frac{R_\pi Z_{c1}}{R_\pi - R_c}\right) \frac{Z_{01} + jZ_{c1} \tan \theta_c}{Z_{c1} + jZ_{01} \tan \theta_c}. \tag{19}$$

Also, the voltages V_1, V_2, V_3 and V_4 are related by transmission line equations and by using Kirchhoff's current law, we have

$$\frac{V_2}{V_1} = \frac{V_4 (Z_{01} + jZ_{c1} \tan \theta_c)}{V_3 (Z_{01} + jZ_{\pi 1} \tan \theta_\pi)} = \frac{Z_2 // Z_{ic}}{Z_1 // Z_{i\pi}} \tag{20}$$

where $(Z_2 // Z_{ic})$ denotes the parallel combination of two impedances Z_2 and Z_{ic}. For perfect isolation, V_3 and V_4 are related as in (14). So, (20) could be solved to get:

$$\begin{aligned}(Z_{01} Z_{02} Z_{c1} &- R_c R_\pi Z_{c1} Z_{\pi 1}^2) \tan \theta_\pi \\ &= (Z_{01} Z_{02} Z_{\pi 1} - R_c R_\pi Z_{\pi 1} Z_{c1}^2) \tan \theta_c\end{aligned} \tag{21}$$

which is the condition for infinite isolation in a general non-symmetrical inhomogeneous directional coupler.

Appendix II
Derivation of the Conditions for Perfect Match and Perfect Isolation

For perfect match at port 1 ($S_{11} = 0$), we require

$$Z_{01} = Z_1 // Z_{i\pi} + Z_2 // Z_{ic} \tag{22}$$

By solving (20) and (22), $Z_{i\pi}$ and Z_{ic} can be expressed as

$$Z_{i\pi} = \frac{V_1 Z_{01} Z_{02}}{Z_{02}(V_1 + V_2) - Z_{01} R_\pi (R_\pi V_1 + R_c V_2)} \tag{23}$$

and

$$Z_{ic} = \frac{V_2 Z_{01} Z_{02}}{Z_{02}(V_1 + V_2) - Z_{01} R_c (R_\pi V_1 + R_c V_2)} \tag{24}$$

Equations (18), (19), (23) and (24) are used to find the following equations

$$\left(\frac{R_c Z_{\pi 1}}{R_c - R_\pi}\right) \frac{Z_{01} + jZ_{\pi 1} \tan \theta_\pi}{Z_{\pi 1} + jZ_{01} \tan \theta_\pi} = \frac{V_1 Z_{01} Z_{02}}{Z_{02}(V_1 + V_2) - Z_{01} R_\pi (R_\pi V_1 + R_c V_2)} \tag{25}$$

and

$$\left(\frac{R_\pi Z_{c1}}{R_\pi - R_c}\right) \frac{Z_{01} + jZ_{c1} \tan \theta_c}{Z_{c1} + jZ_{01} \tan \theta_c} = \frac{V_2 Z_{01} Z_{02}}{Z_{02}(V_1 + V_2) - Z_{01} R_c (R_\pi V_1 + R_c V_2)} \tag{26}$$

If $\tan \theta_\pi$ and $\tan \theta_c$ are large so that $Z_{\pi 1, c1} \tan \theta_{\pi, c} >> Z_{01}$ and $Z_{01} \tan \theta_{\pi, c} >> Z_{\pi 1, c1}$, (25) and (26) can be further simplified as

$$\frac{R_c}{R_c - R_\pi} \left(\frac{Z_{\pi 1}^2}{Z_{01}}\right) = \frac{Z_{01} Z_{02}}{Z_{02}(1 + \mathrm{a}) - R_\pi Z_{01} (R_\pi + R_c \mathrm{a})} \tag{27}$$

and

$$\frac{R_\pi}{R_\pi - R_c} \left(\frac{Z_{c1}^2}{Z_{01}}\right) = \frac{\mathrm{a} Z_{01} Z_{02}}{Z_{02}(1 + \mathrm{a}) - R_c Z_{01} (R_\pi + R_c \mathrm{a})} \tag{28}$$

where parameter a is the ratio of V_2 and V_1, i.e. the ratio of c mode and π mode voltages at port 1 (including the forward and reflected waves). These two equations are used to write Z_{01} and Z_{02} in terms of normal mode parameters and the parameter a as follows:

$$Z_{01}^2 = \frac{(1 + \mathrm{a})Z_{\pi 1}^2 Z_{c1}^2}{Z_{c1}^2 + \mathrm{a} Z_{\pi 1}^2} \tag{29}$$

and

$$Z_{01}Z_{02} = \frac{R_\pi R_c (R_\pi + R_c \mathrm{a}) Z_{\pi 1}^2 Z_{c1}^2}{R_\pi Z_{c1}^2 + \mathrm{a} R_c Z_{\pi 1}^2}. \tag{30}$$

Equations (29) and (30) are derived from the requirements of $S_{31} = 0$ and $S_{11} = 0$. For a perfect coupler, we still need the conditions $S_{24} = 0$ and $S_{44} = 0$ to be met. This gives us another two equations, namely:

$$Z_{02}^2 = \frac{(1 + \mathrm{b}) R_c^2 R_\pi^2 Z_{\pi 1}^2 Z_{c1}^2}{Z_{c1}^2 + \mathrm{b} Z_{\pi 1}^2} \tag{31}$$

and

$$Z_{01}Z_{02} = \frac{R_\pi R_c (R_c + R_\pi \mathrm{b}) Z_{\pi 1}^2 Z_{c1}^2}{R_c Z_{c1}^2 + \mathrm{b} R_\pi Z_{\pi 1}^2} \tag{32}$$

where parameter b is defined as the ratio of c mode voltages at port 4 (including the forward and reflected waves). Equations (29), (30), (31) and (32) are used to solve for four unknowns: a, b, Z_{01} and Z_{02}. The solutions are:

$$\mathrm{a} = \left|\frac{R_\pi}{R_c}\right| \frac{Z_{c1}}{Z_{\pi 1}} \tag{33}$$

$$\mathrm{b} = \left|\frac{R_c}{R_\pi}\right| \frac{Z_{c1}}{Z_{\pi 1}} \tag{34}$$

$$Z_{01} = \sqrt{Z_{c1} Z_{\pi 1} \frac{(|R_c| Z_{\pi 1} + |R_\pi| Z_{c1})}{(|R_c| Z_{c1} + |R_\pi| Z_{\pi 1})}} \tag{35}$$

$$Z_{02} = \sqrt{R_c^2 R_\pi^2 Z_{c1} Z_{\pi 1} \frac{(|R_c| Z_{c1} + |R_\pi| Z_{\pi 1})}{(|R_c| Z_{\pi 1} + |R_\pi| Z_{c1})}}. \tag{36}$$

Equations (33) to (36) are the conditions to be satisfied for good input match and isolation. The coupling from port 1 to port 4 is

$$S_{41} = \frac{R_\pi V_1 + R_c V_2}{V_1 + V_2} \sqrt{\frac{Z_{01}}{Z_{02}}}$$

$$= \frac{R_\pi + R_c \mathrm{a}}{1 + \mathrm{a}} \sqrt{\frac{Z_{01}}{Z_{02}}}$$

$$= \sqrt{\frac{|R_c R_\pi| (Z_{c1} - Z_{\pi 1})^2}{(|R_c| Z_{\pi 1} + |R_\pi| Z_{c1})(|R_c| Z_{c1} + |R_\pi| Z_{\pi 1})}}. \tag{37}$$

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